The GSI Time Anomaly: Facts and Fiction

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The claims that the GSI time anomaly is due to the mixing of neutrinos in the final state of the observed electron-capture processes are refuted. With the help of an analogy with a double-slit experiment, it is shown that the standard method of calculation of the rate of an interaction process by adding the rates of production of all the allowed final states, regardless of a possible coherence among them, is correct. It is a consequence of causality. It is shown that the GSI time anomaly may be caused by quantum beats due to the existence of two coherent energy levels of the decaying ion with an extremely small energy splitting (about $6 \times 10^{-16}\,\text{eV}$) and relative probabilities having a ratio of about $1/99$.

An anomalous oscillatory time modulation of the electron capture decays

$$^{140}\text{Pr}^{58+} \rightarrow {}^{140}\text{Ce}^{58+} + \nu_e \,, \qquad (1)$$

$$^{142}\text{Pm}^{60+} \rightarrow {}^{142}\text{Nd}^{60+} + \nu_e \,. \qquad (2)$$

has been observed in a GSI experiment [1]. The data are fitted by an oscillatory decay rate with a period $T \simeq 7\,\text{s}$ and an amplitude $A \simeq 0.2$.

It has been proposed that the GSI anomaly is due to the interference of the massive neutrinos which compose the final electron neutrino state [1, 2, 3, 4]:

$$|\nu_e\rangle = \cos\vartheta\,|\nu_1\rangle + \sin\vartheta\,|\nu_2\rangle \,, \qquad (3)$$

where ϑ is the solar mixing angle (see Ref. [5]). In order to assess the viability of this explanation of the GSI anomaly, it is necessary to understand the meaning of interference [6].

Interference is the result of the addition (superposition) of two or more waves. If the waves come from the same source, interference can occur if the waves evolve different phases by propagating through different paths.

Let us consider, as an example, the well-known double-slit interference experiment with classical or quantum waves depicted in Fig. 1. In a double slit experiment an incoming plane wave packet hits a barrier with two tiny holes, generating two outgoing spherical wave packets which propagate on the other side of the barrier. The two outgoing
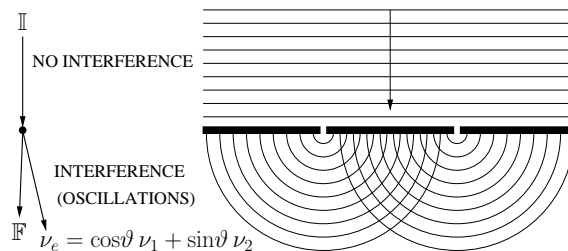

Figure 1. Analogy between the electron-capture decay process (4) and a double-slit interference experiment.

waves are coherent, since they are created with the same initial phase in the two holes. Hence, the intensity after the barrier, which is proportional to the squared modulus of the sum of the two outgoing waves, exhibits interference effects. The interference depends on the different path lengths of the two outgoing spherical waves after the barrier. Here, the important words for our discussion are "after the barrier". The reason is that we can draw an analogy between the double-slit experiment and an electron-capture decay process of the type in Eqs. (1) and (2), which can be schematically written as

$$\mathbb{I} \rightarrow \mathbb{F} + \nu_e \,. \qquad (4)$$

Taking into account the neutrino mixing in

Eq. (3), we have two different decay channels:

$$\mathbb{I} \to \mathbb{F} + \nu_1\,, \qquad \mathbb{I} \to \mathbb{F} + \nu_2\,. \tag{5}$$

The initial state in the two decay channels is the same. In our analogy with the double-slit experiment, the initial state $\mathbb{I}$ is analogous to the incoming wave packet. The two final states $\mathbb{F} + \nu_1$ and $\mathbb{F} + \nu_2$ are analogous to the two outgoing wave packets. The different weights of ν_1 and ν_2 production due to $\vartheta \neq \pi/4$ correspond to different sizes of the two holes in the barrier.

In the analogy, the decay rate of $\mathbb{I}$ corresponds to the fraction of intensity of the incoming wave which crosses the barrier, which depends only on the sizes of the holes. It does not depend on the interference effect which occurs after the wave has passed through the barrier. In a similar way, the decay rate of $\mathbb{I}$ cannot depend on the interference of ν_1 and ν_2 which occurs after the decay has happened.

Of course, flavor neutrino oscillations caused by the interference of ν_1 and ν_2 can occur after the decay, in analogy with the occurrence of interference of the outgoing waves in the double-slit experiment, regardless of the fact that the decay rate is the incoherent sum of the rates of production of ν_1 and ν_2 and the fraction of intensity of the incoming wave which crosses the barrier is the incoherent sum of the fractions of intensity of the incoming wave which pass trough the two holes.

The above argument is a simple consequence of causality: the interference of ν_1 and ν_2 occurring after the decay cannot affect the decay rate.

Causality is explicitly violated in Ref. [2], where the decaying ion is described by a wave packet, but it is claimed that there is a selection of the momenta of the ion caused by a final neutrino momentum splitting due to the mass difference of ν_1 and ν_2. This selection violates causality. In the double-slit analogy, the properties of the outgoing wave packets are determined by the properties of the incoming wave packet, not vice versa. In a correct treatment, all the momentum distribution of the wave packet of the ion contributes to the decay, generating appropriate neutrino wave packets.

The authors of Refs. [3, 4] use a different approach: they calculate the decay rate with the
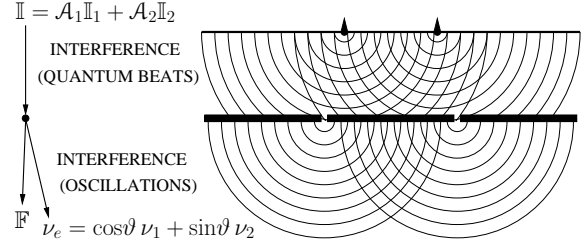

Figure 2. Analogy between quantum beats in the electron-capture decay process (4) and a double-slit interference experiment with two sources.

final neutrino state

$$|\nu\rangle = |\nu_1\rangle + |\nu_2\rangle\,. \tag{6}$$

This state is different from the standard electron neutrino state in Eq. (3). It is not even properly normalized to describe one particle ($\langle\nu|\nu\rangle = 2$). Moreover, it leads to a decay rate which is different from the standard one, given by the incoherent sum of the rates of decay into the different massive neutrinos final states weighted by the corresponding element of the mixing matrix [6]. The analogy with the double-slit experiment and the causality argument discussed above support the correctness of the standard decay rate. As a final argument against the final neutrino state in Eq.(6), one can check that the corresponding decay rate does not reduce to the Standard Model decay rate in the limit of massless neutrinos [6].

Although the GSI time anomaly cannot be due to effects of neutrino mixing in the final state of the electron-capture process, it can be due to interference effects in the initial state. For example, there could be an interference between two coherent energy states of the decaying ion which produces quantum beats. Also in this case we can draw an analogy with a double-slit experiment. However, we must change the setup, considering the double-slit experiment with two coherent sources of incoming waves depicted in Fig. 2. In this case, the two incoming waves interfere at the holes in the barrier, leading to a modulation of the intensity which crosses the barrier. The role of causality is clear: the interference effect is due

to the different phases of the two coherent incoming waves at the holes, which have been developed during the propagation of the two waves along different path lengths before reaching the barrier. Analogously, quantum beats in the GSI experiment can be due to interference of two coherent energy states of the decaying ion which develop different phases before the decay. If the measuring apparatus which monitors the ions with a frequency of the order of the revolution frequency in the ESR storage ring, about 2 MHz, does not distinguish between the two states, their coherence is preserved for a long time.

If the two energy states of the decaying ion $\mathbb{I}_1$ and $\mathbb{I}_2$ are produced at the time $t = 0$ with amplitudes $\mathcal{A}_1$ and $\mathcal{A}_2$ (with $|\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 = 1$), we have

$$|\mathbb{I}(t = 0)\rangle = \mathcal{A}_1 |\mathbb{I}_1\rangle + \mathcal{A}_2 |\mathbb{I}_2\rangle. \tag{7}$$

Assuming, for simplicity, that the two states with energies E_1 and E_2 have the same decay rate Γ, at the time t we have

$$|\mathbb{I}(t)\rangle = \left(\mathcal{A}_1 e^{-iE_1 t} |\mathbb{I}_1\rangle + \mathcal{A}_2 e^{-iE_2 t} |\mathbb{I}_2\rangle\right) e^{-\Gamma t/2}. \tag{8}$$

The probability of electron capture at the time t is given by

$$\begin{aligned} P_{\text{EC}}(t) &= |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}(t)\rangle|^2 \\ &= [1 + A\cos(\Delta E t + \varphi)]\,\overline{P}_{\text{EC}}\,e^{-\Gamma t}. \end{aligned} \tag{9}$$

where where S is the S-matrix operator, $A \equiv 2|\mathcal{A}_1||\mathcal{A}_2|$, $\Delta E \equiv E_2 - E_1$,

$$\overline{P}_{\text{EC}} = |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_1\rangle|^2 = |\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_2\rangle|^2, \tag{10}$$

and φ is a constant phase which takes into account possible phase differences of $\mathcal{A}_1$ and $\mathcal{A}_2$ and of $\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_1\rangle$ and $\langle \nu_e, \mathbb{F}|\mathsf{S}|\mathbb{I}_2\rangle$.

The fit of GSI data presented in Ref. [1] gave

$$\Delta E \simeq 6 \times 10^{-16}\,\text{eV}, \qquad A \simeq 0.2. \tag{11}$$

Therefore, the energy splitting is extremely small. The authors of Ref. [1] noted that the splitting of the two hyperfine $1s$ energy levels of the electron is many order of magnitude too large (and the

contribution to the decay of one of the two states is suppressed by angular momentum conservation). It is difficult to find a mechanism which produces a smaller energy splitting. Furthermore, since the amplitude $A \simeq 0.2$ of the interference is rather small, it is necessary to find a mechanism which generates coherently the states $\mathbb{I}_1$ and $\mathbb{I}_2$ with probabilities $|\mathcal{A}_1|^2$ and $|\mathcal{A}_2|^2$ having a ratio of about 1/99!

In conclusion, I have shown that the standard method of calculation of the rates (cross sections and decay rates) of interaction processes by summing over the rates of production of all the allowed channels with a defined number of particles in the final state, regardless of a possible coherence among them, is correct [6]. The argument has been clarified through an analogy with a double-slit experiment, emphasizing that it is a consequence of causality. I have explained the reasons why the claim in Refs. [2,3,4] that the GSI time anomaly is due to the mixing of neutrinos in the final state of the electron-capture process is incorrect (see also Ref. [7]). I have also shown that the GSI time anomaly may be due to quantum beats due to the existence of two coherent energy levels of the decaying ion. However, since the required energy splitting is extremely small (about 6×10^{-16} eV) and the two energy levels must be produced with relative probabilities having a ratio of about 1/99, finding an appropriate mechanism is very difficult.

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